Homework 9 - Metric and volume form

Q9.1. Use abstract index notation to show that

$$\star \boldsymbol{v} = \boldsymbol{v} \cdot \boldsymbol{\epsilon} \quad \Rightarrow \quad \boldsymbol{v} = \boldsymbol{\epsilon}^{-1} \cdot \star \boldsymbol{v} \tag{Q9.1.1}$$

Q9.2. Show that for an n-form $\boldsymbol{\omega}$ in an N-dimensional space

$$*^{-1}\boldsymbol{\omega} = (-1)^{n(N-n)} \operatorname{sgn}(g) * \boldsymbol{\omega} \tag{Q9.2.1}$$

Q9.3. (a) Express Maxwell's equations in terms of $\underline{E}$ and $\underline{\underline{B}}$.

(b) Use

$$\underline{\underline{G}} = *\underline{\underline{F}} - \underline{N} \tag{Q9.3.1}$$

to express Maxwell's equations in terms of $\underline{\underline{F}}$

(c) Use

$$*^{(4)} \left(\underline{e}^t \wedge \underline{E} \right) = - *^{(3)} \underline{E} \tag{Q9.3.2}$$

and

$$*^{(4)} \underline{\underline{B}} = \underline{e}^t \wedge *^{(3)} \underline{\underline{B}} \tag{Q9.3.3}$$

to show that

$$\underline{N} = -\underline{e}^t \wedge \underline{M} + \underline{\underline{P}} \tag{Q9.3.4}$$

(d) Give the topological and physical meanings of $\underline{\nabla} \wedge \underline{N}$ from both the four dimensional and three plus one dimensional points of view.