

## Homework 2 - Partial differentiation

The derivative  $f'(x)$  of a function  $f(x)$  can be defined by

$$\delta f \equiv f(x + \delta x) - f(x) = f'(x) \delta x + \mathcal{O}((\delta x)^2) \quad (\text{H2.1})$$

In the limit that  $\delta x$  becomes infinitesimal,  $\delta x \rightarrow dx$ , the higher order terms disappear, leaving

$$df = f'(x) dx \quad (\text{H2.2})$$

and hence the notation

$$\frac{df}{dx} \equiv f'(x) \quad (\text{H2.3})$$

For a function  $f(x, y)$  of two variables, the infinitesimal change in the function can be written as

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy \quad (\text{H2.4})$$

where the **partial derivatives**  $\partial f/\partial x$  and  $\partial f/\partial y$  are derivatives with respect to  $x$  at constant  $y$ , and with respect to  $y$  at constant  $x$ , respectively

$$\frac{\partial f}{\partial x} \equiv \left. \frac{df}{dx} \right|_{dy=0} \quad (\text{H2.5})$$

$$\frac{\partial f}{\partial y} \equiv \left. \frac{df}{dy} \right|_{dx=0} \quad (\text{H2.6})$$

For a function  $f(x)$  of position, the infinitesimal change in position is a vector,  $\vec{dx}$ , and so

$$df = \frac{df}{d\vec{x}} \cdot \vec{dx} \quad (\text{H2.7})$$

with the spatial derivative usually expressed with the notation

$$\nabla f \equiv \frac{df}{d\vec{x}} \quad (\text{H2.8})$$

Q2.1. The Lagrangian  $L(q, \dot{q}, t)$  satisfies the Euler-Lagrange equation

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) = \frac{\partial L}{\partial q} \quad (\text{Q2.1.1})$$

where

$$\dot{q} \equiv \frac{dq}{dt} \quad (\text{Q2.1.2})$$

Show that the Hamiltonian

$$H \equiv p\dot{q} - L \quad (\text{Q2.1.3})$$

where

$$p \equiv \frac{\partial L}{\partial \dot{q}} \quad (\text{Q2.1.4})$$

is a function of  $q, p, t$  and not  $\dot{q}$ , i.e.  $H = H(q, p, t)$ , and that

$$\frac{dq}{dt} = \frac{\partial H}{\partial p} \quad (\text{Q2.1.5})$$

$$\frac{dp}{dt} = -\frac{\partial H}{\partial q} \quad (\text{Q2.1.6})$$

and

$$\frac{dH}{dt} = \frac{\partial H}{\partial t} \quad (\text{Q2.1.7})$$