2.5 De Sitter space

2.5.1 De Sitter metric

De Sitter space is the maximally symmetric solution of

$$G_{\mathbf{ab}} = \Lambda g_{\mathbf{ab}} \tag{2.5.1}$$

corresponding to a vacuum energy dominated universe with constant curvature

$$R_{\mathbf{abcd}} = \frac{\Lambda}{3}\left(g_{\mathbf{ad}}g_{\mathbf{bc}} - g_{\mathbf{ac}}g_{\mathbf{bd}}\right) \tag{2.5.2}$$

The de Sitter metric can be expressed in a variety of coordinate systems, corresponding to a spatially spherical, contracting then expanding, universe

$$d\tau^2 = dt^2 - H^{-2}\cosh^2(Ht)\left(d\xi^2 + \sin^2\xi\, d\Omega^2\right) \tag{2.5.3}$$

a spatially flat

$$d\tau^2 = dt^2 - \exp(2Ht)\left(dr^2 + r^2\, d\Omega^2\right) \tag{2.5.4}$$

or hyperbolic

$$d\tau^2 = dt^2 - H^{-2}\sinh^2(Ht)\left(d\xi^2 + \sinh^2\xi\, d\Omega^2\right) \tag{2.5.5}$$

expanding universe, or a static universe showing the **cosmological horizon** at $r = 1/H$

$$d\tau^2 = \left(1 - H^2r^2\right) dt^2 - \left(\frac{dr^2}{1 - H^2r^2} + r^2\, d\Omega^2\right) \tag{2.5.6}$$

where

$$H = \sqrt{\frac{\Lambda}{3}} \tag{2.5.7}$$

and

$$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2 \tag{2.5.8}$$

2.5.2 Penrose diagrams

The global structure of de Sitter space can be illustrated using conformal compactification, which distorts distances but preserves angles and hence the light cone. Starting from the metric in the form of Eq. (2.5.3), which covers all of de Sitter space, conformally rescaling the metric

$$\hat{g}_{\mathbf{ab}} = \left[\frac{H^2}{\cosh^2(Ht)}\right] g_{\mathbf{ab}} \tag{2.5.9}$$

and changing coordinates

$$\eta = \tan^{-1}\sinh(Ht) \tag{2.5.10}$$

gives

$$d\hat{\tau}^2 = d\eta^2 - d\xi^2 - \sin^2\xi\, d\Omega^2 \tag{2.5.11}$$

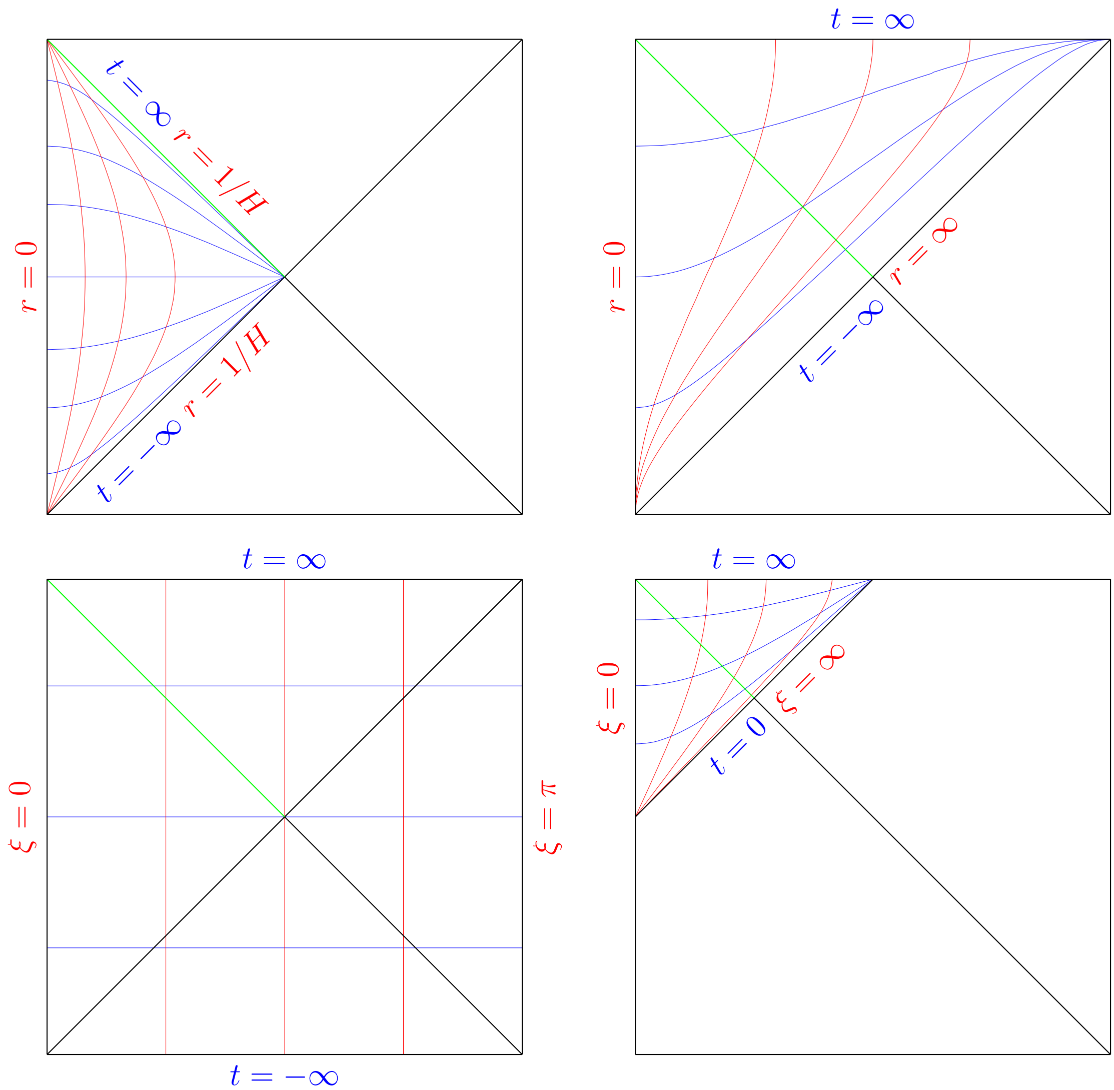

Figure 2.5.1: Penrose diagram for de Sitter spacetime showing the cosmological horizon relative to the origin. Top left: static coordinates; top right: flat coordinates; bottom left: spherical coordinates; bottom right: hyperbolic coordinates.

which has finite extent

$$\eta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \quad , \quad \xi \in [0, \pi] \tag{2.5.12}$$

and can be represented by a **Penrose diagram**, see Figure 2.5.1. The Penrose diagrams for Minkowski and Schwarzschild spacetimes are given in Figures A3.2.1 and 2.6.1.