Homework 5 - Weyl tensor

Q5.1. Show that

$$C_{\mathbf{abc}}{}^{\mathbf{b}} = 0 \qquad \text{(Q5.1.1)}$$

Q5.2. Show that in four dimensional spacetime

$$\nabla_{\mathbf{d}} C_{\mathbf{abc}}{}^{\mathbf{d}} = -\nabla_{[\mathbf{a}} D_{\mathbf{b}]\mathbf{c}} \qquad \text{(Q5.2.1)}$$

where

$$D_{\mathbf{bc}} = R_{\mathbf{bc}} - \frac{1}{6} R g_{\mathbf{bc}} \qquad \text{(Q5.2.2)}$$

Q5.3. Show that

  (a) in one dimension

$$R_{\mathbf{abcd}} = 0 \qquad \text{(Q5.3.1)}$$

  (b) in two or less dimensions

$$G_{\mathbf{ab}} = 0 \qquad \text{(Q5.3.2)}$$

  (c) in three or less dimensions

$$C_{\mathbf{abcd}} = 0 \qquad \text{(Q5.3.3)}$$

and interpret these results physically.