Homework 4 - Schwarzschild metric

Q4.1. You synchronize two clocks and then throw one up into the air and catch it. Which clock will show the earlier time? By how much?

Q4.2. Consider the spacetime with metric

$$d\tau^2 = \left(1 - \frac{2GM}{r} - H^2 r^2\right) dt^2 - \left(1 - \frac{2GM}{r} - H^2 r^2\right)^{-1} dr^2 - r^2 \, d\Omega^2 \quad \text{(Q4.2.1)}$$

    (a) Describe the properties of the spacetime.

    (b) Draw its Penrose diagram.

    (c) Describe the possible circular orbits of a particle moving freely in this spacetime.

Q4.3. A particle of mass m falls towards a Schwarzschild black hole of mass M. The particle is initially a great distance from the black hole and has energy $E = m$ and angular momentum $L \simeq 4GMm$.

    (a) In the case $L > 4GMm$, describe qualitatively its trajectory.

    (b) In the case $L < 4GMm$, describe qualitatively its trajectory.

    (c) In the case $L = 4GMm$, calculate its trajectory $r(\phi)$ and asymptotic orbital period from the point of view of

       i. an astronaut following the trajectory

       ii. a distant observer