Homework 2 - Einstein equation

Q2.1. Use the Bianchi identity

$$\nabla_{\mathbf{a}} R_{\mathbf{bcd}}{}^{\mathbf{e}} + \nabla_{\mathbf{b}} R_{\mathbf{cad}}{}^{\mathbf{e}} + \nabla_{\mathbf{c}} R_{\mathbf{abd}}{}^{\mathbf{e}} = 0 \qquad (Q2.1.1)$$

to show that

$$\nabla_{\mathbf{b}} G_{\mathbf{a}}{}^{\mathbf{b}} = 0 \qquad (Q2.1.2)$$

Q2.2. Calculate

$$\nabla_{\mathbf{b}} T_{\mathbf{a}}{}^{\mathbf{b}} = 0 \qquad (Q2.2.1)$$

for a perfect fluid and interpret your answer.