Homework 1 - Newtonian perspective

Q1.1. For a particle with spacetime velocity $u^{\mathbf{a}}$ experiencing a spacetime force $f_{\mathbf{a}}$, show that

$$f_{\mathbf{a}} u^{\mathbf{a}} = 0 \qquad \text{(Q1.1.1)}$$

and interpret this equation from a Newtonian perspective.

Q1.2. In the space-time decomposition of the metric, Eq. (2.2.9), set

$$A = 1 + 2\phi \qquad \text{(Q1.2.1)}$$

and use the Newtonian approximation

$$\phi \, , \, B_{\mathbf{a}} v^{\mathbf{a}} \, , \, h_{\mathbf{ab}} v^{\mathbf{a}} v^{\mathbf{b}} \ll 1 \qquad \text{(Q1.2.2)}$$

(a) Reexpress the relativistic uncharged particle action in space-time decomposed form and interpret the leading terms in your action.

(b) Calculate the Euler-Lagrange equation for the leading terms in your action and interpret the resulting equation.

(c) Reexpress the spacetime momentum in space-time decomposed form and interpret the leading terms.